NUMBER AND ALGEBRA

ALGEBRA

Algebra is the study of mathematical patterns and rules. The subject can be traced back over 4000 years ago to the ancient Babylonians. Today, all modern technology relies on algebra – the internet, satellites, computers, TVs and all other digital devices rely on the application of algebra.


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Chapter outline

	Working mathematically				
3.01 Variables	U	F		R	C
3.02 From words to algebraic expressions	U	F		R	C
3.03 Substitution	U	F	PS		
3.04 Collecting variables	U	F		R	C
3.05 Adding and subtracting terms	U	F		R	C
3.06 Multiplying terms	U	F		R	C
3.07 Dividing terms	U	F		R	C
3.08 Extension: The index laws*	U	F			
3.09 Expanding expressions	U	F		R	C
3.10 Factorising algebraic terms	U	F		R	
3.11 Factorising expressions	U	F		R	C
3.12 Factorising with negative terms	U	F		R	C
3.13 Extension: Expanding binomial products*	U	F		R	C

***Year 9 Extension, Stage 5.2**

Wordbank

algebraic expression An expression that describes a quantity by using variables and numerals, such as $3x + 4y - 8$

algebraic term A part of an algebraic expression, such as $3x$ in $4x^2 + 3x - 11$

evaluate To find the value of an expression

expand To rewrite an expression such as $4(2k + 4)$ without brackets

factorise The opposite of expand; to rewrite an expression, such as $8k + 16$, with brackets

like terms Terms that have exactly the same variables, for example, $5p$ and $2p$

substitute To replace a variable with a number

variable (or **pronumeral**) A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- learn about variables as a way of representing numbers using letters
- generalise number laws and properties to algebraic terms and expressions
- convert between word and algebraic expressions
- evaluate algebraic expressions by substituting values for each variable
- simplify algebraic expressions using the 4 arithmetic operations
- (EXTENSION, STAGE 5.2) apply index laws to algebraic expressions with whole number and zero powers
- use the distributive law to expand algebraic expressions with grouping symbols (brackets)
- factorise algebraic terms and expressions
- (EXTENSION, STAGE 5.2) expand binomial products

SkillCheck ANSWERS ON P. 549

1 Evaluate each expression.

a $3 - 4$	**b** $-5 + 9$	**c** $-8 - 2$
d $3 - (-2)$	**e** -3×6	**f** $-12 \div (-4)$
g $-12 \div 4 - 10$	**h** $-4 \times (-8)$	**i** $(-6)^2$
j 2^3	**k** $(-1)^3$	**l** 2×3^2

2 Simplify each algebraic expression.

a $7 \times p$	**b** $a \times b$	**c** $k \times k$
d $n \times n \times n$	**e** $2 \times p \times m$	**f** $p \times k \times d$
g $y \times 6$	**h** $t \times 3 \times q$	**i** $6d - 7d$
j $q + q + q$	**k** $a + a + a + a$	**l** $1x$

3 Choose which operation (+, −, × or ÷) is related to each word.

a sum	**b** product	**c** difference	**d** quotient
e decrease	**f** increase	**g** more than	**h** less than

4 If $x = 8$ and $y = 2$, evaluate each expression.

a $x + y$	**b** $3x$	**c** $x - y$	**d** $5y$
e xy	**f** $9 - y$	**g** $x + 17$	**h** $2x + 4$
i $3y - 2$	**j** $x \div y$	**k** $5x \div y$	**l** $4x + 5y$

WS
What does the symbol mean?

5 Find:

a the sum of 8 and 5	**b** double 9
c 6 increased by 10	**d** the difference between 14 and 11
e the product of 3 and 7	**f** how many times 5 divides into 40

Variables

3.01

In algebra, a **variable** or **pronumeral** is a symbol, usually a letter of the alphabet, that stands for a number. It is called a variable because its value can vary (change).

When writing **algebraic expressions**, we use the following abbreviations.

- $3 \times k = 3k$ for multiplication, we leave out the '×' symbol
- $m \div 4 = \frac{m}{4}$ for division, we can write in fraction form
- $r \times r = r^2$ for powers
- $1x = x$ for multiplying by 1
- $c \times a \times 6 = 6ac$ write the number first, then the variables in alphabetical order

Algebraic expressions

Algebra bingo game

3.01

Example 1

Simplify each expression.

a $p + p + p + p$ **b** $d \times 5$ **c** $r \div 7$

d $m \times n \times 6$ **e** $4v + 2v$ **f** $5 \times k \times 2 \times h$

g $y \times y \times y$ **h** $5a - 4a$ **i** $b \times b \times 15$

Solution

a $p + p + p + p = 4p$ 4 lots of p.

b $d \times 5 = 5d$ Write the number first.

c $r \div 7 = \frac{r}{7}$

d $m \times n \times 6 = 6mn$ Number first, then variables in alphabetical order.

e $4v + 2v = 6v$ $(v + v + v + v) + (v + v) = 6$ lots of v

f $5 \times k \times 2 \times h = 5 \times 2 \times k \times h$

$= 10hk$

g $y \times y \times y = y^3$

h $5a - 4a = 1a$ $(a + a + a + a + a) - (a + a + a + a) =$ one a left

$= a$ Just a: no need to write the '1'.

i $b \times b \times 15 = 15b^2$ The '2' (squared) belongs to the b only.

Order of operations

Example 2

Use order of operations to simplify each expression.

a $2 \times m - 4$ **b** $15 - k \div 3$ **c** $13 \div (d \times d) + 4$

Solution

a $2 \times m - 4 = 2m - 4$ — × first, then −

b $15 - k \div 3 = 15 - (k \div 3)$ — ÷ first, then −

$= 15 - \frac{k}{3}$

c $13 \div (d \times d) + 4 = 13 \div d^2 + 4$ — ( ) first, then ÷, then +

$= \frac{13}{d^2} + 4$

Example 3

Write each expression in expanded form.

Note that expanding is the opposite of simplifying.

a $5bc$ **b** $-4kr^2$ **c** $x^2 - \frac{t}{11}$

Solution

a $5bc = 5 \times b \times c$

b $-4kr^2 = -4 \times k \times r \times r$

c $x^2 - \frac{t}{11} = x \times x - t \div 11$

EXERCISE 3.01 ANSWERS ON P. 549

Variables U F R C

EXAMPLE 1

1 Simplify each expression. R C

a $w \times 5$

b $m \times m$

c $3 \times c \times b \times a$

d $k + k + k + k + k + k$

e $m \div 8$

f $f + f + f$

g $12m + 4m$

h $7 \times w \times 3 \times x$

i $11p - 2p$

j $4 \times t \times t$

k $6d - 5d$

l $b \times c \times d \times 16$

m $26 \div n$

n $h \times 9 \times h$

o $9x + 11x$

p $a + a - a$

q $12q - 4q$

r $2 \times c \times 6 \times n$

s $3 \times r \times d \times (-2) \times d$

t $a + a + a + c + c$

u $a \times a \times a \times x \times x$

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2 Use order of operations to simplify each expression. R C

a $13 \div (a \times 2)$ b $9 + 4 \times m$ c $(e - 6) \div 5$

d $4 \times t - 8$ e $y \times y + z \times z$ f $k \div 9 - n$

g $p \div (9 + n)$ h $22 - e \div 2$ i $11 + u \times u \times 11$

EXAMPLE 2

3 Which expression is equal to $m \times m \times 5 + 2$?
Select the correct answer **A**, **B**, **C** or **D**. R C

A $5m^2 + 2$ **B** $5m^3 + 2$ **C** $5m + 2$ **D** $m^2 + 7$

4 Explain the meaning of: C

a pq b $\frac{3y}{x}$ c $1x = x$

5 Which expression is **not** equal to $4p$? Select **A**, **B**, **C** or **D**. R C

A $5p - p$ **B** p^4 **C** $p + p + p + p$ **D** $p \times 4$

6 Write each expression in expanded form. C

EXAMPLE 3

a $8st$ b $-2y^2$ c $\frac{8}{f}$ d $r^2 + t^2$

e $4b^2 d$ f $5xy - 2a$ g $\frac{x-2}{3}$ h q^3

i $16 - 2d^2$ j $\frac{-4d}{3}$ k $h^2 i^2$ l $4j - \frac{9}{d}$

7 What is $7e^3$ in expanded form? Select **A**, **B**, **C** or **D**. C

A $7e \times 7e \times 7e$ **B** $7e + 7e + 7e$ **C** $7 \times e + e + e$ **D** $7 \times e \times e \times e$

Investigation

The laws of arithmetic

We can use algebraic symbols to describe general laws about arithmetic.

For example, if we add zero to any number, the answer is still that number.

If we let N stand for any number: $N + 0 = N$

In groups of 2 to 4, answer the following questions.

1 Describe in words what each number rule below means.

a $N \times 1 = N$ b $N \times 0 = 0$

c $a - b \neq b - a$ d $a + b + c = b + a + c$

e $N - 0 = N$ f $ab = ba$

2 Write each rule algebraically using variables.

a Any number divided by 1 equals itself.

b Multiplying a number by 8 is the same as doubling it 3 times.

c Any 3 numbers can be multiplied together in any order.

d Any number added to itself is the same as multiplying that number by 2.

e Any number subtracted from itself equals 0.

f Any number multiplied by its reciprocal equals 1.

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3 Is each equation true or false? Test your answer by substituting a number for each variable and checking.

a $a \div b = b \div a$

b $N \div N = 1$

c $4a - a = 4$

d a is a factor of a

e If N is even, then $N + 3$ is odd

f $\frac{1}{2}N = N - 2$

g $a + (-a) = 2a$

h $0 \div N = 0$

i $N \div 0 = 0$

j $N \div 1 = N$

4 If k is an odd number, what is an expression for:

a the previous odd number?

b the next even number?

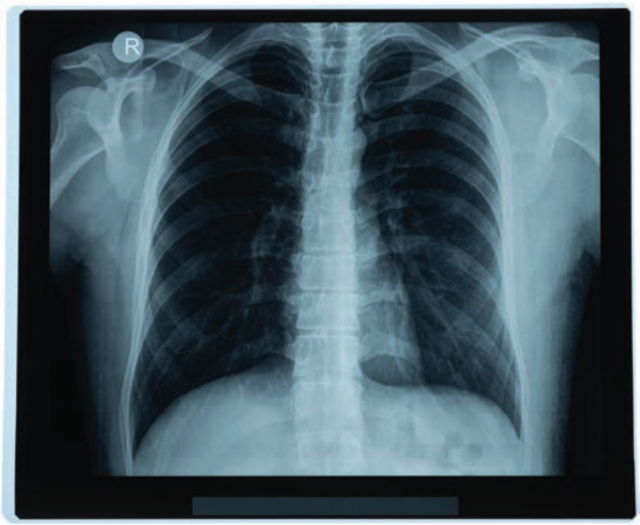
Did you know?

The X factor

In algebra the letter '*x*' is used quite frequently as a variable. During the 9th century, the Arabic word ***al-jabr*** ('algebra') described the process of moving variables from one side of an equation to the other to find the value of an unknown. The word for 'thing' or 'object' (the unknown number) in Arabic was ***shei***, translated into Greek as ***xei***, and shortened to ***x***. ***Xenos*** is also the Greek word for unknown, stranger or foreigner, which could explain why early European mathematicians used the letter ***x*** in algebra.

One further explanation is that ancient scholars could only write using feathers dipped in ink. The two easiest letters to write were ***x*** and ***y***. Mathematicians needed variables they could write quickly without mixing them up. As a result, *x* and *y* are still the two most commonly used variables in algebra.

Find examples of X-words that involve something unknown, such as X-ray, Brand X or X factor.



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From words to algebraic expressions

3.02

From words to algebraic expressions

Algebraic expressions

WS Writing algebraic expressions

Generalised arithmetic

What does the symbol mean?

Example 4

If x is a variable representing any number, write an algebraic expression for:

a 4 times the number

b the number divided by 6

c the previous consecutive number

d one-third of the number

consecutive means 'following in order': for example, 7, 8, 9 are consecutive numbers

Solution

a $4 \times x = 4x$

b $x \div 6$ or $\frac{x}{6}$

c $x - 1$

d $\frac{1}{3}x = \frac{x}{3}$

The number before x is $x - 1$ (for example, the number before 4 is $4 - 1 = 3$).

Example 5

Think of a number, y, multiply it by 3 and subtract 7.
Write an algebraic expression for the answer.

Solution

First, we multiply the number y by 3 to get $y \times 3 = 3y$.

Then, we subtract 7 from our result to get the answer, $3y - 7$.

Example 6

Describe the algebraic expression $\frac{a+b}{2}$ in words.

Solution

$\frac{a+b}{2}$ is the sum of a and b, divided by 2, or the average of a and b.

EXERCISE 3.02 ANSWERS ON P. 549

From words to algebraic expressions U F R C

EXAMPLE 4

1 If n represents any number, write an algebraic expression for: R C

a 5 times the number
b the number plus 6
c the number divided by 10
d the next consecutive number
e half of the number
f 20 less than the number
g the number multiplied by itself
h 20 minus the number
i the previous consecutive number
j double the number
k 6 divided by the number
l the difference between the number and 8
m the product of the number and (−4)
n the square root of the number

2 If a, b and c represent any 3 numbers, write an algebraic expression for: R C

a the sum of a, b and c
b the product of a and b
c the difference between b and c, where b is greater than c
d the product of a and c, plus b
e 3 times b, plus a
f c divided by b
g how much a is greater than b
h 5 more than (b times c)
i c less than b
j a times b divided by c
k the square root of the sum of a and c
l the product of a and b, decreased by 7 times c

3 A cricketer scored a total of x runs in 6 innings. What was her average score? Select the correct answer **A, B, C** or **D**. R C

A $6x$
B $6 \div x$
C x^6
D $x \div 6$


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4 Write an algebraic expression for the answer to each instruction. **C**

EXAMPLE 5

- **a** Think of a number (y), multiply it by 3 and add 8
- **b** Think of a number (p), divide it by 3 and then subtract 6
- **c** Think of a number (M), multiply it by 4 and then add 27
- **d** Think of a number (k), multiply it by 5 and then divide it by 2
- **e** Think of a number (x), divide it by 6 and add 13
- **f** Think of a number (B), halve it and then subtract 2
- **g** Think of a number (R), divide it by 9 and then add 10
- **h** Think of a number (n), multiply it by 7 and then multiply it by 3

5 Describe each algebraic expression in words. **C**

EXAMPLE 6

a $x + y - z$	**b** $5K + L$	**c** $xy - 3$	**d** $\frac{W}{5}$
e $3x - 1$	**f** $\frac{y-x}{10}$	**g** $2N + 21$	**h** $7 - e$

6 Write an expression for: **R** **C**

- **a** the number of girls in a class if there are b boys and a total of t students
- **b** the number of bottles of drink needed for a party if each bottle can be shared by 5 guests and there are n guests
- **c** the amount of change in dollars that Sandra receives if she paid for 3 hamburgers costing \$$p$ each with \$$y$ in cash
- **d** the perimeter of an isosceles triangle with equal sides of length r and an uneven side of length 9
- **e** the cost of one egg if a carton of a dozen eggs costs \$$a$
- **f** the perimeter of a square of length x
- **g** the number of rows of chairs needed to place 160 chairs in the hall if each row contains r chairs

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- **h** the total cost of attending the pool for x adults and y children, where admission is \$5 per adult and \$3 per child

3.02

3.03 Substitution

Substitution

Formula 1 game

Substitution game

WS Substitution

The word **substitute** means replacing one thing with another. In algebra, substitution means to replace a variable with a value to evaluate an algebraic expression.

Example 7

If $p = 3$ and $q = 14$, evaluate each expression.

a $2p + q$ **b** $\frac{pq+2}{4}$ **c** $5q - p^2$

Solution

a $2p+q = 2\times3+14$
$= 20$

b $\frac{pq+2}{4} = \frac{3\times14+2}{4}$
$= \frac{44}{4}$
$= 11$

c $5q - p^2 = 5\times14 - 3^2$
$= 61$

Substituting into formulas

A **formula** is a general rule that is written as an algebraic equation and shows the relationship between variables. Solving mathematical problems often involves substituting values into formulas.

Example 8

The formula for the perimeter of a rectangle is $P = 2l + 2w$, where P is the perimeter, l is the length and w is the width. Use this formula to calculate the perimeter of a rectangle with length 12 m and width 9.5 m.

Solution

When length $l = 12$ and width $w = 9.5$,

$P = 2l + 2w$
$= 2\times12 + 2\times9.5$
$= 43$ m

EXERCISE 3.03 ANSWERS ON P. 550

Substitution U F PS

EXAMPLE 7

1 If $x = 5$ and $y = 6$, what is the value of $2x - y$? Select the correct answer **A**, **B**, **C** or **D**.

A 19 **B** -2 **C** 4 **D** 16

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2 If $a = 3$, $b = 5$ and $c = 2$, evaluate each expression.

a $a + b$	**b** $4a$	**c** c^2	**d** $a + b + c$
e abc	**f** $3b - a$	**g** $c + a - b$	**h** $4a - 2c$
i $10 - b$	**j** $ab + c^2$	**k** $c - b$	**l** $\frac{2a}{c}$

3 If $d = 10$, $e = -3$ and $f = 5$, evaluate each expression.

a def	**b** f^2	**c** e^3	**d** $d \div f$
e $de \div 2$	**f** $d^2 - f^2$	**g** $7e$	**h** $3e + 2d - f$
i $f + d - e$	**j** $\frac{d}{2} - 3f$	**k** $6ef$	**l** $\frac{e+f}{4}$

4 If $p = -3$, what is the value of $2p^2$? Select **A**, **B**, **C** or **D**.

A 36	**B** 18	**C** −18	**D** −36

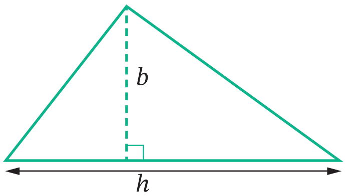
5 If $y = -1$ and $z = 4$, evaluate each expression.

a $3 - 4z$	**b** $-3y^2$	**c** $5(z - 2)$	**d** $y(z + 1)$
e $\frac{y}{z}$	**f** $\frac{z}{4}$	**g** $\frac{y+z}{9}$	**h** $\frac{10}{3+y}$

6 If $n = 6$, which of the following expressions is equal to 22? Select **A**, **B**, **C** or **D**.

A $\frac{n+4}{2}$	**B** $\frac{2n+4}{2}$	**C** $\frac{n^2}{2}+4$	**D** $\frac{n^2+4}{2}$

7 The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A is the area of the triangle, b is the length of the base and h is the height.



Find the area of a triangle with base length 12 m and height 7 m.

EXAMPLE 8

8 A plumber charges according to the formula $C = 42h + 65$, where C is the charge in dollars and h is the number of hours worked. How much does the plumber charge for working for $3\frac{1}{2}$ hours?

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9 The formula for converting Celsius temperatures to Fahrenheit temperatures is $F = \frac{9}{5}C + 32$, where F is the temperature in Fahrenheit and C is the temperature in Celsius. Use the formula to convert 26°C to Fahrenheit. PS

10 The number of hours of sleep recommended for children has the formula $H = 17 - \frac{A}{2}$, where H is the number of hours and A is the age of the child.
Find the number of hours of sleep recommended for a 6-year-old. PS

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11 The formula for the cost of a party is $C = 45n + 500$, where C is the cost in dollars and n is the number of guests. Use this formula to calculate the cost of a party for 120 guests. PS

12 The time (t minutes) taken to travel to school depends on how far the student walks and how far the student travels on the bus. The formula for the time taken is $t = 11w + 4b$, where w is the walking distance and b is the distance travelled by bus.

a Find the time taken for each student to travel to school: PS

i Michael, who walks 1 km and goes 5 km on the bus

ii Vittoria, who walks 4 km and goes 3 km on the bus

b If school starts at 9 a.m., what is the latest time Wendy can leave home if she needs to walk 2 km and go on the bus for 7 km and still reach school on time?

Technology

Substitution

1 Enter the variables and values shown into a new spreadsheet.

	A	B
1	**variable**	**value**
2	m	10
3	n	12
4	p	4
5	q	0
6	r	2.7
7	s	-8
8	t	25
9	u	-5

This means m = **10.**

2 In cell D1, enter the formula **=B4-B9** to evaluate $p - u$. You should get 9 because $4 - (-5) = 9$.

3 In cell D2, enter the formula **=B2+B3** to evaluate $m + n$. You should get 22 because $10 + 12 = 22$.

4 For each cell below, enter a formula to evaluate the given expression.

D3: pr

D4: $\frac{t}{u}$

D5: $n + t - p$

D6: $n - \frac{m}{u}$

D7: $\frac{n-m}{u}$

D8: $\frac{s}{p} \times r$

D9: p^3

D10: $(s + n) \times (p + r)$

D11: $t + pq + s$

D12: $\sqrt{t}$

D13: $mnpq$

D14: the average of r, s, t and u

D15: $\frac{6m-n}{p+s}$

D16: $\sqrt{n+p}$

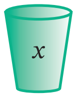
D17: ts^2

D18: $n^3 - r^2$

Collecting variables

3.04

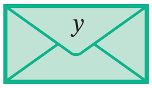
Let the variable x stand for an unknown number. Let x be represented by a cup that holds the unknown number of balls.



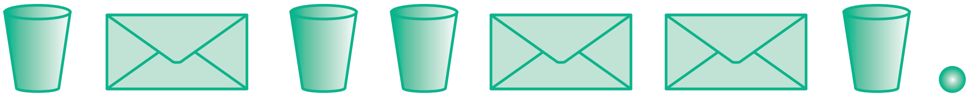
Then the total number of balls represented in the diagram below can be written using the algebraic expression $3 \times x + 5 = 3x + 5$.

Note that even though we don't know how many balls are in each cup, it is the same number of balls, x, and we can still describe the total number of balls algebraically.

Let y stand for an unknown number that is different from x. Let y be represented by an envelope that contains this number of balls.



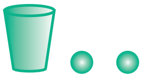
Then the total number of balls represented in the diagram below can be written using the algebraic expression $4x + 3y + 1$.



EXERCISE 3.04 ANSWERS ON P. 550

Collecting variables U F R C

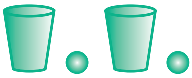
1 Write an algebraic expression for the number of balls represented in each diagram, if each cup holds x balls. R C

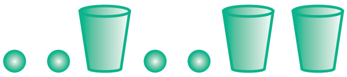
a 

b 

c 

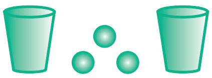
d 

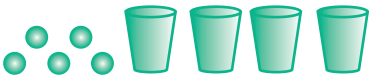
e 

f 

g 

h 

i 

j 

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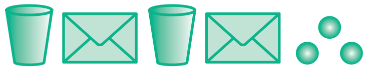
2 Write an algebraic expression for the number of balls represented in each diagram, if each cup holds x balls and each envelope holds y balls. **R** **C**

a 

b 

c 

d 

e 

f 

g 

h 

3 Draw cups and balls to represent each algebraic expression, if each cup holds x balls. **C**

a $x + 3$
b $x + x + 1$
c $4 + 2x$
d $2 + x + 2$
e $x + 5 + 2x$
f $1 + 2x + 3 + x$

4 If N stands for the number of paperclips in each envelope, which diagram represents $3N + 2$ paperclips? Select the correct answer **A**, **B**, **C** or **D**. **R** **C**

A 

B 

C 

D 

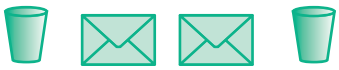
5 If m stands for the number of coins in each envelope and p stands for the number of coins in each cup, match each expression in the left-hand column with the correct expression in the right-hand column. **R** **C**

a 

$m + p + m$

A 

$2m + 2p$

b 

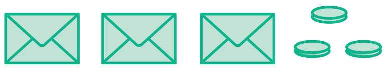
$p + m + m + p$

B 

$2p + 3$

c 

$p + m + p + m + p$

C 

$3m + 3$

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d

$p + 2 + p + 1$

D

$p + 2m$

e

$2m + 2 + m + 1$

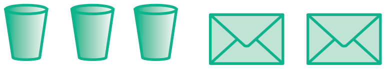
E

$3p + 4$

f

$2 + p + 2 + 2p$

F



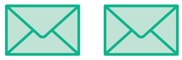
$3p + 2m$

6 Match each expression in the left column with the correct expression on the right. **R** **C**

a

$m + p - m$

A



$2m$

b

$m + 4 - 2$

B

$2p + 2$

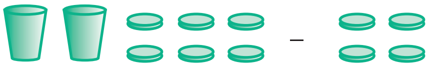
c

$2m + p - p$

C

$2p$

d



$2p + 6 - 4$

D

p

e

$2m + 2p - 2m$

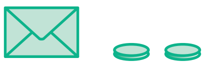
E

$2p + 2m$

f

$2p + 3m - m$

F



$m + 2$

7 Simplify each expression. **R** **C**

a

$p + m + p$

b

$m + 2p - p$

c

$2p + 2 + p + 3$

d 

$p + m + 2 + p + m - 1$

e 

$3p + 2m - m - p$

8 Simplify $9a + 4b - 5a + 2b$. Select **A**, **B**, **C** or **D**. **R**

A $14a + 6b$ **B** $10ab$ **C** $4a + 6b$ **D** $14a + 11b$

9 Simplify each expression. **R**

a $3d + 2d + 4a$ **b** $6h + 4r + 5r - h$ **c** $e + e - e$

d $2k + 3 + 2k - 6$ **e** $3x + 4z - x + 2z$ **f** $2s - p - s + 3p$

g $6u + 4 - 2u + 8$ **h** $7n - 6 + 5 - 3n$ **i** $20 + 6i - 3i - 20$

Mental skills 3A: Maths without calculators ANSWERS ON P. 550

Simplifying multiplication by factorising

Sometimes, one big multiplication may be made into many simpler little multiplications if we break one or both terms into 2 ‘easier’ factors. Then we use the property that numbers can be multiplied in any order.

1 Study each example.

a $24 \times 8 = 6 \times 4 \times 2 \times 4$
$= 4 \times 4 \times 6 \times 2$
$= 16 \times 6 \times 2$
$= 96 \times 2$
$= 192$

Look for pairs of numbers that you can multiply easily and rearrange

b $15 \times 6 = 3 \times 5 \times 6$
$= 3 \times 30$
$= 90$

c $14 \times 36 = 7 \times 2 \times 6 \times 6$
$= 6 \times 7 \times 6 \times 2$
$= 42 \times 6 \times 2$
$= 252 \times 2$
$= 504$

d $30 \times 22 = 10 \times 3 \times 22$
$= 10 \times 66$
$= 660$

Note: For each question, there are many different possible ways of arriving at the correct result.

2 Now evaluate each product by factorising first.

a 30×24 **b** 18×27 **c** 25×33 **d** 16×8

e 28×20 **f** 12×18 **g** 21×9 **h** 36×12

i 16×35 **j** 22×28 **k** 9×15 **l** 27×25

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Adding and subtracting terms 3.05

What is the fastest way to mentally calculate the sum 3 + 3 + 5 + 3 + 5 + 5 + 3 + 3 + 5?

Do you think of collecting all the 3s together and all the 5s together like this?

$$
\begin{aligned}
3+3+5+3+5+5+3+3+5 &= (3+3+3+3+3)+(5+5+5+5) \\
&= (5\times 3)+(4\times 5) \\
&= 15+20 \\
&= 35
\end{aligned}
$$

We can do the same thing algebraically. Suppose an unknown number p is represented by a cup  and another unknown number m is represented by an envelope .
Now consider these 2 sums.

a

$p + m + p + m = 2p + 2m$

b

$m + p + m + p + m = 3m + 2p$

Note that we can add all the p **terms** together, and we can add all the m terms together, but we cannot add the ps and ms together because p and m represent different numbers. We cannot simplify the answers $2p + 2m$ and $3m + 2p$ because p and m are different.

These are examples of collecting **like terms**, terms that contain exactly the same variable(s), such as $4h$, $5h$ and h. Here are some more examples.

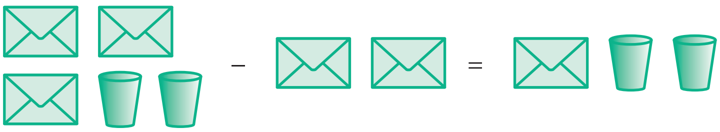
$3x$ and $-5x$	$4mw$ and $2mw$	$12m$ and m
$-6d$ and $10d$	$5ab$ and $2ba$	xyz and $2yzx$

Note: $ab = ba$

Here are some examples of **unlike terms**, terms that have different variables.

$3x$ and $5m$	$4mw$ and $2mn$	$-6d$ and $10d^2$

We can add or subtract like terms because the variables in them represent the same number.
We can only add or subtract things that are the same. For example:



$3m + 2p - 2m = m + 2p$

$$
\begin{aligned}
(3 \text{ lots of } m)+(2 \text{ lots of } p)-(2 \text{ lots of } m) &= (3 \text{ lots of } m - 2 \text{ lots of } m)+(2 \text{ lots of } p) \\
&= (1 \text{ lot of } m)+(2 \text{ lots of } p)
\end{aligned}
$$

This makes sense because if $m = 5$ and $p = 8$, then

$$
\begin{aligned}
3m+2p-2m &= (3\times 5)+(2\times 8)-(2\times 5) \\
&= (3\times 5)-(2\times 5)+(2\times 8) && \text{grouping like terms} \\
&= (1\times 5)+(2\times 8) && \text{3 lots of 5} - \text{2 lots of 5} = \text{1 lot of 5}
\end{aligned}
$$

Algebra using diagrams

Algebra bingo game

Adding and subtracting like terms

Adding and subtracting terms

When **adding and subtracting terms**, only **like terms** can be collected.

Like terms have exactly the same variables.

Example 9

Select the like terms from each list below.

a $5x, -x, x^2, xy, \frac{1}{2}x, -8x, \frac{4x}{3}$ **b** $3ab, 5, a, ba, -2b, 4ab^2, -ab$

Solution

a $5x, -x, \frac{1}{2}x, -8x, \frac{4x}{3}$ are like terms because they each have a single x in them.

b $3ab$, ba and $-ab$ are like terms because each contains ab.

Adding and subtracting terms

Collecting like terms

Example 10

Simplify each expression by collecting like terms.

a $7d + 3e + 3d + e$ **b** $6m - 8 + 3m + 6$

c $6xy + 7x^2 - 3yx - 12x^2$ **d** $14 - 4t - 9t + 2$

Solution

a $7d + 3e + 3d + e = 7d + 3d + 3e + e$ Grouping like terms

$= 10d + 4e$

b $6m - 8 + 3m + 6 = 6m + 3m - 8 + 6$ The + or − sign goes with the term after it.

$= 9m - 2$

c $6xy + 7x^2 - 3yx - 12x^2 = 6xy - 3xy + 7x^2 - 12x^2$ $3yx = 3xy$

$= 3xy - 5x^2$

d $14 - 4t - 9t + 2 = 14 + 2 - 4t - 9t$

$= 16 - 13t$

EXERCISE 3.05 ANSWERS ON P. 550

Adding and subtracting terms U F R C

EXAMPLE 9

1 Select the like terms from each list. C

a $4p, 2y, 2p, 5z, p$ **b** $m^2, m, n, 3m^2, mn$ **c** $5ac, 4a, 7ca, 5$

d $vw, 5v, 9v, w, v^2, 2wv$ **e** $p, 6pq, 2qp, 3pq, 7q$ **f** $2d, 3d^3, 7d^2, 9, 3d$

g $4mn, 3m, 4, 2nm, mn, 2n$ **h** $x^2y, 2x, 3y, 4x^2y$ **i** $7, 2a, 4b, 5ba, ab$

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2 What are the like terms in $3ab$, $2a^2$, $2b^2a$, $2p^2$, $2ba$?
Select the correct answer **A**, **B**, **C** or **D**. **C**

A $2a^2, 2p^2$ **B** $2a^2, 2b^2a, 2p^2, 2ba$ **C** $3ab, 2ba$ **D** $2b^2a, 2ba$

3 Simplify each expression by collecting like terms.

a $4k + 7k$ **b** $5mn + 2nm$ **c** $xy + xy$
d $3abc + 4abc + 2bac$ **e** $3x^2 + 2x^2 + x^2$ **f** $ef^2 + 4ef^2$
g $8d - 3d$ **h** $12mk - 7mk$ **i** $12xy - 7yx - xy$
j $8de - 12de$ **k** $6k^2 - 3k^2$ **l** $4w^2 - w^2$

3.05

EXAMPLE 10

4 Which expression is $2m + 4p - 5m - 2p$ simplified? Select **A**, **B**, **C** or **D**.

A $-mp$ **B** $7m + 6p$ **C** $-3m + 2p$ **D** $-3p$

5 Simplify each expression.

a $5k - 2j + 6j$ **b** $10ab + 2 - ab$ **c** $12m^2 - 5m^2 + 2m$
d $12s^2 - 7st - 2s^2$ **e** $4mn + 7mn - mn$ **f** $14k + 3 - 5k$
g $y^2 + 2y^2 - y^2$ **h** $9d - 4e + 3d$ **i** $7 - 2x - 3$
j $15gb - 8gb + gb$ **k** $4 - 2q + 5$ **l** $5a + 6c - 4c$

6 Simplify each expression.

a $4x - y - x - 2y$ **b** $7ab + 2bc - 3ab + bc$
c $4k^2 + 3 + 5k^2 + 8$ **d** $4bc - 8a + 2bc - a$
e $x^2 - 6x - x + 7$ **f** $p^2 + q^2 + 3p^2 - q^2$
g $3 + 4y + 8y + 11$ **h** $11r - 3s - 4r + 6s$
i $7g + 8b - b - g$ **j** $6x^2 - 4x - 9x + 6$
k $20a + 11m + 32a - 11m$ **l** $3q - 7 - 8q + 12q$
m $5 - 2y - 5 - 4y$ **n** $s^2 + 4s^2 + 12 - 5s^2$

7 Which expression is $a^5 + a^5$ simplified? Select **A**, **B**, **C** or **D**.

A a^{10} **B** a^{25} **C** $2a^5$ **D** $2a^{10}$

8 Write an algebraic expression involving a sum that simplifies to: **R** **C**

a $2x + 6$ **b** $6m + 12p$ **c** $x^2 - x$

9 Write an algebraic expression involving a difference that simplifies to: **R** **C**

a $2x + 6$ **b** $6m + 12p$ **c** $x^2 - x$

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3.06 Multiplying terms

WS Perimeter and area

What is the fastest way to mentally calculate the product $6 \times 4 \times 5 \times 3$?

Do you think of rearranging the numbers to pair convenient factors like this?

$$\begin{aligned} 6 \times 4 \times 5 \times 3 &= (4 \times 5) \times (6 \times 3) \\ &= 20 \times 18 \\ &= 360 \end{aligned}$$

We can do the same thing algebraically. As numbers can be multiplied in any order, we can multiply like *and* unlike terms.

Multiplying terms

Example 11

Simplify each product.

a $7b \times 3a$ **b** $8x \times (-2y)$ **c** $5d \times 6d \times 4$

d $9m^2 \times 4p$ **e** $4a \times 3b \times (-2a)$

Solution

a
$$\begin{aligned} 7b \times 3a &= 7 \times b \times 3 \times a \\ &= 7 \times 3 \times a \times b \\ &= 21ab \end{aligned}$$

Group numbers and variables separately.

b
$$\begin{aligned} 8x \times (-2y) &= 8 \times (-2) \times x \times y \\ &= -16xy \end{aligned}$$

c
$$\begin{aligned} 5d \times 6d \times 4 &= 5 \times 6 \times 4 \times d \times d \\ &= 120d^2 \end{aligned}$$

d
$$\begin{aligned} 9m^2 \times 4p &= 9 \times 4 \times m^2 \times p \\ &= 36m^2p \end{aligned}$$

e
$$\begin{aligned} 4a \times 3b \times (-2a) &= 4 \times 3 \times (-2) \times a \times a \times b \\ &= -24a^2b \end{aligned}$$

Multiplying terms

When multiplying algebraic terms, multiply the numbers first, then multiply the variables. Write all variables in alphabetical order.

Multiplying terms

Example 12

Use index notation to simplify each expression.

a $k \times k \times k \times k$ **b** $j \times j \times j \times j \times j \times j$

Solution

a $k \times k \times k \times k = k^4$ **b** $j \times j \times j \times j \times j \times j = j^6$

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EXERCISE 3.06 ANSWERS ON P. 550

Multiplying terms **U F R C**

EXAMPLE 11

3.06

1 Simplify $2p \times 5pq$. Select the correct answer **A**, **B**, **C** or **D**.

A $10pq$ **B** $7p^2q$ **C** $7pq$ **D** $10p^2q$

2 Simplify each expression.

a $5 \times 3y$ **b** $8 \times 2m$ **c** $4k \times m$

d $4t \times 5$ **e** $2 \times f \times 6$ **f** $2x \times 8y$

g $10p \times 3m$ **h** $3b \times 6d$ **i** $3d \times 4d$

j $5 \times 4m \times 6n$ **k** $2y \times 3y \times 8$ **l** $3r \times 3 \times 3r$

m $2 \times 4a \times 9b$ **n** $x \times xy \times y$ **o** $2j \times 3k \times 7$

p $5rst \times 2rs$ **q** $3ac \times 6cd$ **r** $hjk \times hj \times jk$

s $4q^2 \times 6p^2$ **t** $2m \times 3n \times mn$

3 Simplify each expression.

a $-4b \times 2d$ **b** $6m \times (-4m)$ **c** $-2 \times 5k \times 9l$

d $20 \times (-3p)$ **e** $9k \times (-7)$ **f** $-15b \times (-5b)$

g $-3m \times mn \times 4n$ **h** $6abc \times (-2ab)$ **i** $5r \times (-2r) \times (-4)$

j $9d \times (-4e) \times 6d$ **k** $-8x \times 9x \times (-2)$ **l** $5m \times 4 \times (-2m)$

4 Which expression simplifies to $14a^2n^2$? Select **A**, **B**, **C** or **D**.

A $2a \times an \times 7$ **B** $2a \times 7n \times an$ **C** $7a^2 \times 2n$ **D** $7a^2n^2 \times 2a^2n^2$

EXAMPLE 12

5 Simplify each expression.

a $y \times y \times y \times y$ **b** $a \times a \times a$

c $2 \times c \times c$ **d** $t \times t \times t \times t \times t$

e $q \times q \times q \times 7$ **f** $k \times k \times k \times l \times l$

g $m \times m \times m \times n \times n$ **h** $-5 \times p \times p \times p \times p \times k \times k \times k$

i $-3 \times y \times y \times y$ **j** $d \times d \times (-1) \times e \times e \times f$

k $q \times q \times r \times r \times r \times r \times 4$ **l** $3p + 2 \times 4p$

m $10m \times 4n - 5mn$ **n** $7a^2 - 2a \times 3a$

o $6r \times 2y - y \times 10r$ **p** $5p - 4p \times 8$

6 Write an algebraic expression involving a product that simplifies to: **R** **C**

a $18de$ **b** $-20a^2y$ **c** $5pr^2$

3.07 Dividing terms

WS Simplifying algebraic expressions

WS Why aren't they the same?

Dividing terms

Dividing terms

When dividing algebraic terms, divide the numbers first, then divide the variables.

Write all variables in alphabetical order.

Example 13

Simplify each quotient.

a $20p \div 5$ **b** $\frac{24y}{8y}$ **c** $\frac{-2m}{4mn}$ **d** $12a^2b \div 3a$

Solution

Group numbers and variables separately.

a $20p \div 5 = \left(\frac{20}{5}\right)p$

$= 4p$

b $\frac{24y}{8y} = \left(\frac{24}{8}\right)\frac{\cancel{y}}{\cancel{y}}$

$= 3$

c $\frac{-2m}{4mn} = \left(\frac{-2}{4}\right)\frac{\cancel{m}}{\cancel{m}n}$

$= \left(\frac{-1}{2}\right)\frac{1}{n}$

$= -\frac{1}{2n}$

d $12a^2b \div 3a = \left(\frac{12}{3}\right)\frac{\cancel{a}ab}{\cancel{a}}$

$= 4ab$

EXERCISE 3.07 ANSWERS ON P. 550

Dividing terms U F R C

EXAMPLE 13

1 Simplify $\frac{55m^2n}{5m}$. Select the correct answer **A**, **B**, **C** or **D**.

A $11mn$ **B** $11m^2n$ **C** $50mn$ **D** $11n$

2 Simplify each quotient.

a $12m \div 3$ **b** $36e \div 4$ **c** $21p \div (-3)$

d $\frac{81x}{9}$ **e** $\frac{60w}{15}$ **f** $\frac{8m}{m}$

g $-25k \div 5k$ **h** $30y \div 40y$ **i** $28pq \div 14q$

j $\frac{63a}{-9a}$ **k** $\frac{4pq}{20q}$ **l** $\frac{-56mn}{8m^2}$

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3 Simplify each quotient.

a $14m^2 \div 7$ **b** $6e^2 \div (-2e)$ **c** $-5p^2 \div p$

d $\frac{14x^2}{2x}$ **e** $\frac{6wm}{-3m}$ **f** $\frac{8m^2n}{2mn}$

g $32k^2y \div (-4k)$ **h** $9jk^2 \div 12jk^2$ **i** $\frac{15d^2e^2}{3de}$

j $\frac{3a^2b^2c}{3ab^2c}$ **k** $\frac{-24acb}{3bc}$ **l** $\frac{-10m^2n}{5mn}$

4 Simplify $12x \div (-3) \times 2x$. Select **A**, **B**, **C** or **D**.

A $-2x$ **B** -2 **C** $-8x$ **D** $-8x^2$

5 Simplify each expression.

a $-18pq \div 6pq$ **b** $48p \div (-8) \div 2p$ **c** $y \times 3y \times (-5y)$

d $9d \times 4e \div 6d$ **e** $8x \times 9x \div 12$ **f** $48m^2 \div 12m \times 2n$

g $\frac{15c}{5c}$ **h** $\frac{a \times 3ab}{ab}$ **i** $\frac{2ab}{10bc}$

j $\frac{3d \times 4e}{6e}$ **k** $\frac{15mn^2p}{3m^2n^2}$ **l** $\frac{-4x^2y \times 3y}{2xy \times (-6xy)}$

m $6x + 4x \div 2$ **n** $14r \div r + 9r$ **o** $24p - \frac{18p^2}{3p}$

6 Write an algebraic expression involving a quotient that simplifies to: **R** **C**

a $7m$ **b** $-3ab$ **c** $5pr^2$

7 Simplify:

a $6w^2 - \frac{10w^5}{5w^3}$ **b** $x^4 - \frac{x^7}{x^3}$

Investigation

Equivalent expressions

State whether each statement below is true or false, then check by substituting a number for the variable and testing whether the left-hand side equals the right-hand side.

a $6 + 4m + 4 = 4m + 10$ **b** $5x + 3x = 8x^2$

c $4k - 7k = -3k$ **d** $7g + 8h - h - g = 7h + 6g$

e $3 + 7n + 13 + 2n = 25n$ **f** $3p \times 7q = 21pq$

g $21m \div 3m = 7m$ **h** $5y - 4y = y$

i $\frac{-16a}{4} = -4a$

3.08 Extension: The index laws

YEAR 9 STAGE 5.2

Indices puzzle

Powers, indices and exponents

Simplifying with the index laws

Index laws

In the previous chapter, *Working with numbers*, we discovered properties about powers, called **index laws**.

Here are some examples:

- $2^3 \times 2^4 = 2^{3+4} = 2^7$
- $7^5 \times 7 = 7^{5+1} = 7^6$
- $4^6 \div 4^2 = 4^{6-2} = 4^4$
- $\frac{10^3}{10^2} = 10^{3-2} = 10^1$
- $(3^4)^2 = 3^{4\times 2} = 3^8$
- $(8^5)^3 = 8^{5\times 3} = 8^{15}$
- $5^0 = 1$
- $8^0 = 1$

Now we can write these index laws algebraically and use them to simplify algebraic expressions.

Index laws

When **multiplying terms with powers** of the same base, **add** the powers.

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with powers** of the same base, **subtract** the powers.

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

When **raising a term with a power** to another power, **multiply** the powers.

$$(a^m)^n = a^{m\times n}$$

Any number raised to the **power of zero** equals **1**.

$$a^0 = 1$$

Now we can apply these index laws algebraically to variables.

Example 14

Simplify each expression.

a $p^6 \times p^2$ **b** $3m^2n \times 7m^3n^5$ **c** $a^{12} \div a^4$ **d** $\frac{42y^6}{7y^2}$

e $(d^4)^5$ **f** $(4m^2)^3$ **g** $2k^0$ **h** $(2k)^0$

Solution

a $p^6 \times p^2 = p^{6+2}$ Adding the powers

$= p^8$

b $3m^2n \times 7m^3n^5 = 3 \times 7 \times m^{2+3} \times n^{1+5}$
$= 21m^5n^6$

c $a^{12} \div a^4 = a^{12-4}$ — Subtracting the powers
$= a^8$

d $\dfrac{42y^6}{7y^2} = \dfrac{42}{7} \times y^{6-2}$
$= 6y^4$

e $(d^4)^5 = d^{4\times5} = d^{20}$ — Multiplying the powers

f $(4m^2)^3 = 4^3 \times (m^2)^3$ — because $(4m^2)^3 = 4m^2 \times 4m^2 \times 4m^2$
$= 64m^{2\times3}$
$= 64m^6$

g $2k^0 = 2 \times k^0$ — The zero power belongs to k only
$= 2 \times 1$
$= 2$

h $(2k)^0 = 1$ — The zero power belongs to $(2k)$

EXERCISE 3.08 ANSWERS ON P. 551

Extension: The index laws U F

EXAMPLE 14

1 Simplify $a^2b \times a^4b^3$. Select the correct answer **A**, **B**, **C** or **D**.

A a^8b^3 **B** a^8b^4 **C** a^6b^3 **D** a^6b^4

2 Simplify each product.

a $m^5 \times m^2$ **b** $k^8 \times k$ **c** $y^4 \times y^6$ **d** $x^7 \times x$
e $n^4 \times n^4$ **f** $q^2 \times q^8$ **g** $b^4 \times b^{10} \times b^2$ **h** $s^3 \times s^5 \times s^7$
i $6k^3 \times 7k^3$ **j** $5y^3 \times 4y$ **k** $d^4 \times 3d^5$ **l** $11e^3 \times 5e$
m $h^4n^2 \times h^6n^5$ **n** $10c^3d^4 \times (-3c^3d)$ **o** $f^5g^4 \times f^3g^7$ **p** $-5p^3q \times 2p^4q$

3 Simplify $\dfrac{36d^6e^4}{12d^3e^4}$. Select **A**, **B**, **C** or **D**.

A $3d^3$ **B** $3d^3e$ **C** $3d^2e$ **D** $3d^2$

4 Simplify each quotient.

a $y^6 \div y^5$ **b** $p^4 \div p$ **c** $\dfrac{q^5}{q^2}$ **d** $n^8 \div n^3$
e $\dfrac{k^{11}}{k^9}$ **f** $x^7 \div x^5$ **g** $r^8 \div r^8$ **h** $m^{12} \div m^5 \div m^2$
i $q^5 \times q^3 \div q^7$ **j** $45g^6 \div 5g^2$ **k** $18p^7 \div 6p^3$ **l** $a^{12} \div a^6 \div a$
m $66x^4y^2 \div 6x^3y$ **n** $\dfrac{x^4 \times x^2}{x^3}$ **o** $\dfrac{70k^5l^6}{10k^2l}$ **p** $\dfrac{15e^3f}{21ef^2}$

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YEAR 9 STAGE 5.2

5 Simplify $(a^4b^2)^3$. Select **A**, **B**, **C** or **D**.

A a^7b^5 **B** $a^{12}b^6$ **C** $a^{12}b^5$ **D** a^7b^6

6 Simplify each expression.

a $(y^8)^2$ **b** $(x^2)^5$ **c** $(y^3)^3$ **d** $(m^2)^9$

e $(k^7)^3$ **f** $(t^{20})^2$ **g** $(3m)^2$ **h** $(2m^4)^3$

i $(xy)^3$ **j** $(p^3q)^4$ **k** $(3h^4)^4$ **l** $(k^3z)^5$

m $(2kj)^4$ **n** $(4c^3d)^2$ **o** $(5x^2y^8)^3$ **p** $(2m^4p^2)^6$

7 Simplify each expression.

a y^0 **b** $(xy)^0$ **c** $(2r - r)^0$ **d** $5u^0$

e $\left(\frac{x}{2}\right)^0$ **f** $(2k^7)^0$ **g** $m^8 \div m^8$ **h** $h^2 \div (h^0)^3$

i $(b^3)^2 \div 2b^0$ **j** $(3x)^0 \times (4x)^2$ **k** $(d^0)^5 \times (d^8)^0$ **l** $26x^4y^3 \div 2x^3y^3$

m $4m^0 + 3m - 3k^0$ **n** $\frac{27b^3d}{9bd}$ **o** $(8h)^2 \times 2h^0 \div 4h^2$ **p** $\frac{6ef^4}{12ef^4}$

8 Simplify each expression.

a $\frac{3pq}{9p^2q^2}$ **b** $\frac{2m^3n^3}{m^2n^2}$ **c** $\frac{8xy}{12xy^2}$

Mental skills 3B Maths without calculators

ANSWERS ON P. 551

Multiplying by 9, 11, 99 or 101

We can use expanding when multiplying by a number near 10 or near 100.

1 Study each example.

a
$$\begin{aligned}25\times11&=25\times(10+1)\\&=25\times10+25\times1\\&=250+25\\&=275\end{aligned}$$

b
$$\begin{aligned}14\times9&=14\times(10-1)\\&=14\times10-14\times1\\&=140-14\\&=126\end{aligned}$$

c
$$\begin{aligned}32\times12&=32\times(10+2)\\&=32\times10+32\times2\\&=320+64\\&=384\end{aligned}$$

d
$$\begin{aligned}7\times99&=7\times(100-1)\\&=7\times100-7\times1\\&=700-7\\&=693\end{aligned}$$

e
$$\begin{aligned}27\times101&=27\times(100+1)\\&=27\times100+27\times1\\&=2700+27\\&=2727\end{aligned}$$

f
$$\begin{aligned}18\times8&=18\times(10-2)\\&=18\times10-18\times2\\&=180-36\\&=144\end{aligned}$$

Foundation Standard Complex

2 Now evaluate each product.

a	16×11	**b**	33×11	**c**	29×9	**d**	45×9
e	62×11	**f**	7×101	**g**	18×101	**h**	36×99
i	19×8	**j**	45×12	**k**	21×102	**l**	6×98

Expanding expressions 3.09

3.08

Algebra using diagrams

Expanding brackets

Expandominoes

Expanding

Expand the brackets

Expand negative brackets

The **distributive law** says that you can multiply by a number by splitting it into the sum or difference of 2 other numbers. Look at these 2 examples.

a $23 \times 12 = 23 \times (10 + 2)$

$= (10 + 2) + (10 + 2) + (10 + 2) + \ldots$ 23 times

$= 10 + 10 + 10 + \ldots + 2 + 2 + 2 + \ldots$ 23 times each

$= (23 \times 10) + (23 \times 2)$

$= 230 + 46$

$= 276$

so $23 \times (10 + 2) = 23 \times 10 + 23 \times 2$

b $35 \times 9 = 35 \times (10 - 1)$

$= (10 - 1) + (10 - 1) + (10 - 1) + \ldots$ 35 times

$= 10 + 10 + 10 + \ldots - 1 - 1 - 1 -$ 35 times each

$= (35 \times 10) - (35 \times 1)$

$= 350 - 35$

$= 315$

so $35 \times (10 - 1) = 35 \times 10 - 35 \times 1$

The distributive law for expanding algebraic expressions

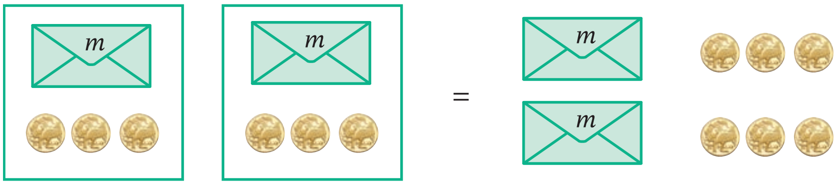
If a, b and c stand for numbers, then: $a(b + c)$ means $a \times (b + c)$

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

We can apply the distributive law to algebraic expressions. Suppose an unknown number m is represented by an envelope  that contains m coins.

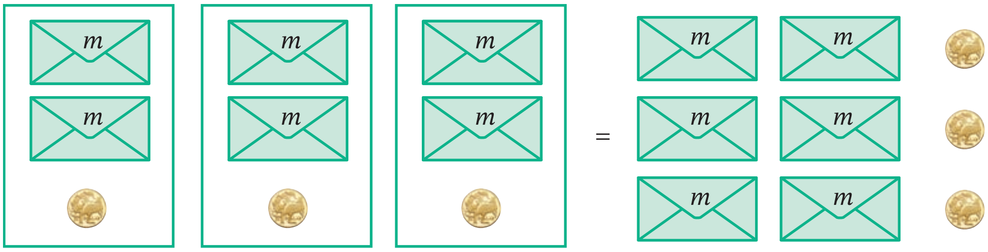
Then $2(m + 3)$ or '2 lots of $(m + 3)$', can be represented by this diagram.



It can be seen that $2(m + 3) = 2 \times m + 2 \times 3 = 2m + 6$.

The number outside the brackets is *multiplied* by each term inside.

Here is another example for $3(2m + 1)$.



It can be seen that $3(2m + 1) = 3 \times 2m + 3 \times 1 = 6m + 3$.

Removing the brackets in an algebraic expression by multiplying each term inside by the term outside is called **expanding** the expression.

Expanding expressions

Example 15

Expand each expression.

a $7(p + 4)$ **b** $3(x - 5)$ **c** $5(2m - 7)$

d $-2(a + 3)$ **e** $-(4d - 2)$ **f** $w(9w - 1)$

Solution

a $7(p + 4) = 7 \times p + 7 \times 4$
$= 7p + 28$

7 is multiplied by each term inside the brackets separately.

b $3(x - 5) = 3 \times x - 3 \times 5$
$= 3x - 15$

c $5(2m - 7) = 5 \times 2m - 5 \times 7$
$= 10m - 35$

d $-2(a + 3) = -2 \times a + (-2) \times 3$
$= -2a + (-6)$
$= -2a - 6$

-2 is multiplied by each term inside the brackets separately.

e $-(4d - 2) = -1(4d - 2)$
$= -1 \times 4d - (-1) \times 2$
$= -4d + 2$

$-(4d - 2)$ is the same as $-1 \times (4d - 2)$.

f $w(9w - 1) = w \times 9w - w \times 1$
$= 9w^2 - w$

Example 16

Expand and simplify each expression.

a $2(x + 5) + 6(x - 4)$ **b** $9k - 4(k - 3)$ **c** $3p - (p + 6)$

Solution

a $2(x+5)+6(x-4) = 2\times x + 2\times 5 + 6\times x + 6\times(-4)$ Expanding

$= 2x + 10 + 6x - 24$

$= 8x - 14$ Collecting like terms

b $9k - 4(k-3) = 9k - 4\times k - 4\times(-3)$

$= 9k - 4k + 12$

$= 5k + 12$

c $3p - (p+6) = 3p - 1(p+6)$

$= 3p - 1\times p - 1\times 6$

$= 3p - p - 6$

$= 2p - 6$

EXERCISE 3.09 ANSWERS ON P. 551

Expanding expressions U F R C

3.09

EXAMPLE 15

1 Expand each expression.

a	$5(m + 3)$	**b**	$8(m - 2)$	**c**	$2(j + 10)$	**d**	$3(a + 2)$
e	$4(h - 7)$	**f**	$6(x + y)$	**g**	$9(4 - b)$	**h**	$11(y - 6)$
i	$4(2k + 5)$	**j**	$3(5m - 2)$	**k**	$6(3x + 4)$	**l**	$10(2a + 5b)$

2 Expand each expression.

a	$-3(n + 4)$	**b**	$-4(v - 5)$	**c**	$-7(d + 3)$	**d**	$-10(k - 2)$
e	$-2(d + e)$	**f**	$-5(5 - z)$	**g**	$-6(h + 1)$	**h**	$-(m - 1)$
i	$-9(5 + 2x)$	**j**	$-(2f + 3)$	**k**	$-3(u - v)$	**l**	$-8(4 - k)$

3 Expand $-3(a - 2b)$. Select the correct answer **A**, **B**, **C** or **D**.

A $-3a - 2b$ **B** $-3a + 6b$ **C** $-3a - 6b$ **D** $-3a + 5b$

4 Expand each expression.

a	$x(y + 6)$	**b**	$p(q - 2)$	**c**	$a(a + 4)$	**d**	$b(3b - 1)$
e	$d(d + 3e)$	**f**	$2(4m + 5n)$	**g**	$5k(3k - 2)$	**h**	$7j(3j + ik)$
i	$x(4x + y)$	**j**	$-2(4p - 3q)$	**k**	$k(k - 12)$	**l**	$-m(4 - m)$
m	$4n(2n + 5m)$	**n**	$2x(1 - 7x)$	**o**	$-f(2f + 4)$	**p**	$-(5j - 7k)$

5 Expand $3y(y - 4)$. Select **A**, **B**, **C** or **D**.

A $3y + 12$ **B** $3y^2 - 7y$ **C** $3y^2 - 12y$ **D** $3y + 7$

Foundation Standard Complex

6 Explain the difference between $2a + 1$ and $2(a + 1)$. **R** **C**

7 Scott says that, to find the perimeter of a rectangle, you double its length, double its width, and add the answers together. Thomas says that you add the length and width first, then double the answer. Who is correct? Can you write each of their answers algebraically? **R** **C**

EXAMPLE 16

8 Expand $12 - 8(x + 2)$. Select **A**, **B**, **C** or **D**.

A $4(x + 2)$ **B** $4x + 8$ **C** $-8x + 14$ **D** $-8x - 4$

9 Expand and simplify each expression.

a $6(a + 2) + 14$ **b** $4k + 2(k + 3)$ **c** $3(q - 3m) + 5m$
d $8 + 6(p - 5)$ **e** $18 - (2y + 14)$ **f** $31 - 7(t + 2)$
g $6(2h - 1) + 11$ **h** $4x - 4(3x - 2)$ **i** $2(9n - 2) + 8$

10 Check that $8(2b - 3) = 16b - 24$ by substituting any value for b and showing that the answers to both sides are equal. **R**

11 Expand and simplify each expression.

a $3(a + 4) + 2(a + 1)$ **b** $2(x + 5) + 6(x - 1)$ **c** $7(4 + k) + 5(2k - 2)$
d $5(2t + 1) + 3(4t + 2)$ **e** $3(n + 5) - 2(n + 4)$ **f** $10(d + 2) - 6(d + 3)$
g $8(4y + 1) - 5(2y - 3)$ **h** $3(p - 4) - 5(p - 1)$ **i** $m(m + 1) + 3(m + 1)$
j $4(d + e) + 3(d - e)$ **k** $2f(3f + 4) - 4(2f - 1)$ **l** $5e(1 - 7e) - e(3e + 4)$

12 Copy and complete each equation. **R**

a $2(\_\_\_\_\_) = 2x + 10$ **b** $4(\_\_\_\_\_) = 4r - 12$ **c** $3(\_\_\_\_\_) = 24k - 9$
d $5(\_\_\_\_\_) = 10a + 35$ **e** $-2(\_\_\_\_\_) = -2y - 18$ **f** $-3(\_\_\_\_\_) = -6t + 21$

Did you know?

Algebraic notation

Algebraic notation is the mathematical language used to describe patterns and rules. It has its own grammar and terminology. For example, the expression $3x^4 - 5x + 7$ has the following features:

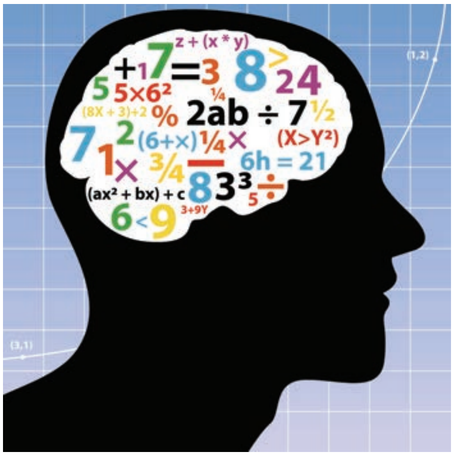
exponent: power or index, 4

coefficient: a number that multiplies a variable, 3 and -5

term: group of numbers and variables, $3x^4$, $-5x$, 7

operator: addition or subtraction

constant: the term with no variable, 7



Dreamstime.com/Beaniebeagle

For the expression $-2x^6 + x^3 - 4$, find the:

a **exponent(s)** **b** **coefficient of x^6** **c** **term containing x^3** **d** **constant**

Foundation Standard Complex

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Factorising algebraic terms

3.10

The **factors** of a number are those numbers that divide into it evenly.
For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.

Algebraic terms also have factors. The **factors** of an algebraic term are those numbers or terms that divide into it evenly. For example, $10xy = 2 \times 5 \times x \times y$, so *some* of the factors of $10xy$ are 2, 5, x, y, 10, $2y$ and $10x$. Often there are too many to list them all easily.

Algebra using diagrams

Factorising using diagrams

3.10

Example 17

List 4 factors of $40a^2b$.

Solution

There are many ways of multiplying terms to get $40a^2b$, such as:

$8 \times 5 \times a \times a \times b$

$4 \times 10 \times a^2 \times b$

$40 \times a^2 \times b$

So 4 factors of $40a^2b$ are: 4, 5, a and a^2b. (*Note*: It is impractical to list them all.)

The highest common factor of algebraic terms

The **highest common factor (HCF)** (or **greatest common divisor (GCD)**) of 2 numbers is the largest number that divides into both numbers evenly. For example, the HCF of 18 and 30 is 6.

The **highest common factor** of 2 algebraic terms is the largest term that divides into both terms evenly. For example, the HCF of $18ab$ and $30b$ is $6b$.

Highest common factor

To find the **highest common factor (HCF)** of 2 or more algebraic terms:

- find the HCF of the numbers
- find the HCF of the variables
- multiply them together to make the highest common algebraic factor.

Example 18

Find the highest common factor of each pair of terms.

a $16x$ and $40xy$ **b** $12r$ and 28

Solution

a First, find the HCF of the numbers.

The factors of 16 are **1**, **2**, **4**, **8**, 16.

The factors of 40 are **1**, **2**, **4**, 5, **8**, 10, 20, 40.

So the HCF of 16 and 40 is **8**.

Then find the HCF of the variables:

The HCF of x and xy is x.

∴ The HCF of $16x$ and $40xy$ is $8 \times x = 8x$.

b First, find the HCF of the numbers:

The factors of 12 are **1**, **2**, 3, **4**, 6, 12.

The factors of 28 are **1**, **2**, **4**, 7, 14, 28.

So the HCF of 12 and 28 is **4**.

Then find the HCF of the variables:

The second term (28) has no variable so the HCF of the variables is 1.

∴ The HCF of $12r$ and 28 is $4 \times 1 = 4$.

EXERCISE 3.10 ANSWERS ON P. 551

Factorising algebraic terms U F R

1 List the factors of each number.

a 15 **b** 8 **c** 17
d 25 **e** 55 **f** 42

2 Find the highest common factor of each pair of numbers. R

a 9 and 6 **b** 6 and 14 **c** 18 and 24
d 16 and 12 **e** 33 and 22 **f** 15 and 25
g 21 and 9 **h** 45 and 30 **i** 16 and 48

EXAMPLE 17

3 List 4 factors of each term.

a $5m$ **b** $7pq$ **c** $10x$ **d** $18d$
e $4k^2$ **f** $21bc$ **g** $32x^2y$ **h** $15ab$

4 Which term is **NOT** a factor of $36m^2n$? Select the correct answer **A**, **B**, **C** or **D**.

A $4m$ **B** $9n$ **C** $8mn$ **D** $12m^2n$

EXAMPLE 18

5 Find the highest common factor of each pair of terms. R

a $3xy$ and $12x$ **b** $4j$ and $24j$ **c** $6x$ and $4ax$
d $21kj$ and $14k$ **e** 5 and $10p$ **f** $16m$ and $12m^2$
g $48cd$ and $32bc$ **h** de and de^2 **i** 9 and $27x^2$
j $3m$ and $6n$ **k** $22m$ and $4mn$ **l** $16y$ and $4y$
m a^2x and ay **n** $64j$ and 32 **o** $15w^2$ and $20wy$
p $22x^2y$ and $36xy^2$ **q** cd and ad **r** $36t$ and $12ty^2$

6 Which term is the highest common factor of $8x^2$ and $12xy$? Select **A**, **B**, **C** or **D**.

A 4 **B** $2x$ **C** $4x$ **D** $24x^2y$

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Factorising expressions

3.11

Expanding an algebraic expression means removing its brackets by multiplying terms.

Factorising is the opposite of expanding. To factorise an algebraic expression, find the HCF of all terms and insert brackets.

expanding

$3(x + 6) \;=\; 3x + 18$

factorising

Algebra using diagrams
Factorising using diagrams
Factorising
3.11

Factorising algebraic expressions

- Find the HCF of all terms and write it outside the brackets
- Divide each term by the HCF and write them inside the brackets
- $ab + ac = a(b + c)$
- $ab - ac = a(b - c)$

To check that the answer is correct, expand it.

Example 19

Factorising expressions

Factorise each expression.

a $2x + 6$ **b** $7 - 7y$ **c** $3ab - 5b$ **d** $r^2 + 7rs$

Solution

a $2x + 6 = 2 \times x + 2 \times 3$
$= 2(x + 3)$

HCF of $2x$ and 6 is 2.
Think: $2 \times$ ___ $= 2x$, $2 \times$ ___ $= 6$
Check by expanding: $2(x + 3) = 2x + 6$

b $7 - 7y = 7 \times 1 - 7 \times y$
$= 7(1 - y)$

HCF of 7 and $7y$ is 7.
Think: $7 \times$ ___ $= 7$, $7 \times$ ___ $= 7y$
Check by expanding: $7(1 - y) = 7 - 7y$

c $3ab - 5b = b \times 3a - b \times 5$
$= b(3a - 5)$

HCF of $3ab$ and $5b$ is b.
Think: $b \times$ ___ $= 3ab$, $b \times$ ___ $= 5b$
Check by expanding: $b(3a - 5) = 3ab - 5b$

d $r^2 + 7rs = r \times r + r \times 7s$
$= r(r + 7s)$

HCF of r^2 and $7rs$ is r.
Think: $r \times$ ___ $= r^2$, $r \times$ ___ $= 7rs$
Check by expanding: $r(r + 7s) = r^2 + 7rs$

EXERCISE 3.11 ANSWERS ON P. 551

Factorising expressions U F R C

1 Find the highest common factor of the terms in each expression. R C

a $10a + 2b$ **b** $4 - 12x$ **c** $ax + xy$ **d** $y^2 - y$

2 Copy and complete each factorisation. R C

a $2x + 4 = 2(\_\_\_\_\_)$ **b** $4p - 16 = 4(\_\_\_\_\_)$

c $12k - 15 = 3(\_\_\_\_\_\_)$ **d** $5j - 20k = 5(\_\_\_\_\_\_)$

e $3x^2 + 3y^2 = 3(\_\_\_\_\_)$ **f** $5x - 15 = \_\_\_\_\_\_(x - 3)$

g $12x - 6 = \_\_\_\_\_\_(2x - 1)$ **h** $2a + 8b = \_\_\_\_\_\_(a + 4b)$

i $4pq + 10 = \_\_\_\_\_\_(2pq + 5)$ **j** $5 + 15x = 5(\_\_\_\_\_)$

k $xy + xz = x(\_\_\_\_\_)$ **l** $ab - ac = a(\_\_\_\_\_)$

m $pq - p = p(\_\_\_\_)$ **n** $cd + bc = c(\_\_\_\_)$

o $a^2 + 2a = a(\_\_\_\_\_\_)$ **p** $5hk - 8h = \_\_\_\_\_\_\_(5k - 8)$

q $24mn + 11n = \_\_\_\_\_\_(24m + 11)$ **r** $xy + y^2 = \_\_\_\_\_\_\_(x + y)$

s $3k^2 + 4jk = \_\_\_\_\_\_(3k + 4j)$ **t** $ef - 2f^2 = \_\_\_\_\_\_\_\_\_\_(e - 2f)$

EXAMPLE 19

3 Factorise each expression. R C

a $4x + 8$ **b** $3m + 6$ **c** $5a - 10$ **d** $15m - 10a$

e $8x - 4y$ **f** $12m - 16w$ **g** $hk - h$ **h** $2xy - xm$

i $12p + 5pr$ **j** $x^2 + 5x$ **k** $4m - m^2$ **l** $y^2 - 11y$

4 Factorise $16k^2 + 18k$. Select the correct answer **A**, **B**, **C** or **D**.

A $2(8k^2 + 9k)$ **B** $k(16k + 18)$ **C** $2k(8k + 9)$ **D** $4k(4k + 9)$

5 Copy and complete each factorisation. R C

a $2am + 2an = 2a(\_\_\_\_\_\_)$ **b** $6xy - 3x = 3x(\_\_\_\_\_)$

c $4ap + 2a = 2a(\_\_\_\_\_\_)$ **d** $5mn - 5mp = 5m(\_\_\_\_\_)$

e $3xy - 12x = 3x(\_\_\_\_\_\_\_)$ **f** $2pq + 2pr = 2p(\_\_\_\_\_\_)$

g $14x - 2xy = 2x(\_\_\_\_\_\_\_)$ **h** $20kj + 5j = 5j(\_\_\_\_\_\_\_)$

i $9xy - 12yz = 3y(\_\_\_\_\_\_\_)$

6 Factorise $3d^2 + 21d$. Select **A**, **B**, **C** or **D**.

A $3(d^2 + 7d)$ **B** $d(3d + 21)$ **C** $d(3d + 7)$ **D** $3d(d + 7)$

7 Factorise each expression. R C

a $7ab + 14a$ **b** $2x - 6xy$ **c** $3m - 9mw$ **d** $10xy - 25xw$

e $8am - 12mp$ **f** $9xy + 6my$ **g** $3x + 9xy$ **h** $ab + abc$

i $xyz + xya$ **j** $18m - 24mn$ **k** $2ac + 3abc$ **l** $pqr + mpq$

m $rs + 7r$ **n** $4s^2 + 16$ **o** $p^2 - 2p$ **p** $6k^3 - 2k$

q $3j + j^2$ **r** $7n - n^2$ **s** $7e + 21e^2$ **t** $6q - 12q^2$

u $a^4 + a^2$ **v** $x^2y + xy^2$ **w** $3m^2n + 9m$ **x** $12a^2b^2c^2 + 3abc$

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Factorising with negative terms

3.12

Factorising with negative terms

When factorising an algebraic expression whose first term is *negative*, include the negative sign in the factor.

$$-ab - ac = -a(b + c)$$
$$-ab + ac = -a(b - c)$$

Algebra using diagrams

Factorising using diagrams

WS Algebra review

WS Algebraic expressions review

Factorising expressions

Expandominoes

Factorising with negative terms

Example 20

Factorise each expression.

a $-5x - 10$ **b** $-9n + 21$ **c** $-gh - h$ **d** $-mn + n^2$

Solution

a $-5x - 10 = -5 \times x + (-5) \times 2$

$= -5(x + 2)$

HCF of $-5x$ and -10 is -5.

Think: $-5 \times$ ___ $= -5x$, $-5 \times$ ___ $= -10$

Check by expanding: $-5(x + 2) = -5x - 10$

b $-9n + 21 = -3 \times 3n + (-3) \times (-7)$

$= -3(3n + (-7))$

$= -3(3n - 7)$

HCF of $-9n$ and 21 is -3.

Think: $-3 \times$ ___ $= -9n$, $-3 \times$ ___ $= 21$

Check by expanding: $-3(3n - 7) = -9n + 21$

c $-gh - h = -h \times g + (-h) \times 1$

$= -h(g + 1)$

HCF of $-gh$ and $-h$ is $-h$.

Think: $-h \times$ ___ $= -gh$, $-h \times$ ___ $= -h$

Check by expanding: $-h(g + 1) = -gh - h$

d $-mn + n^2 = -n \times m + (-n) \times (-n)$

$= -n(m + (-n))$

$= -n(m - n)$

HCF of $-mn$ and n^2 is $-n$.

Think: $-n \times$ ___ $= -mn$, $-n \times$ ___ $= n^2$

Check by expanding: $-n(m - n) = -mn + n^2$

EXERCISE 3.12 ANSWERS ON P. 551

Factorising with negative terms U F R C

1 Copy and complete each factorisation. R C

a $-6k - 6 = -6($________$)$

b $-mn - mp = -m($__________$)$

c $-5r + 10 = -5($_______$)$

d $-3k + 27 = -3($__________$)$

e $-ac + ad =$ _________$(c - d)$

f $-y^2 - 7y =$ ________$(y + 7)$

g $-12y - 30 =$ ________$(2y + 5)$

h $-4f + f^2 =$ ________$(4 - f)$

EXAMPLE 20

2 Factorise each expression. R C

a	$-4x + 12$	**b**	$-3m - 9$	**c**	$-10x + 20$	**d**	$-15x + 10m$
e	$-6x - 8m$	**f**	$-5x - 30$	**g**	$-a - ab$	**h**	$-4xyz + 8abc$
i	$-20 + 4x$	**j**	$-dx - dy$	**k**	$-2m + 5mp$	**l**	$-24x - 16y$

3 Factorise $-14wy - 28wz$. Select the correct answer **A**, **B**, **C** or **D**.

A $-2(7wy + 14wz)$ **B** $-14w(y + 2z)$ **C** $14w(-y - 2z)$ **D** $-2w(7y + 14z)$

4 Factorise each expression. R C

a	$-16xy - 24xk$	**b**	$-20x + 12xy$	**c**	$-8mw + 12m$
d	$5e - 15ef$	**e**	$-ab - abc$	**f**	$xya + xyw$
g	$-12c - 18cw$	**h**	$-2rt + 7rst$	**i**	$20fh + 5h$
j	$-19bw - 38w$	**k**	$-abc + abcd$	**l**	$-abc - abcd$

5 **a** Substitute $w = 2$ into the expressions $-w^2 + 7w$ and $-w(w - 7)$.

b Is $-w^2 + 7w = -w(w - 7)$ for any value of w? R C

6 Factorise $-ab + a^2$. Select **A**, **B**, **C** or **D**.

A $-a(b - a)$ **B** $a(-b - a)$ **C** $-a(b + a)$ **D** $-a(b + a^2)$

7 Factorise each expression. R C

a $a^4bc - a^3b^2c$ **b** $2\pi r^2 + 2\pi rh$

3.13 Extension: Expanding binomial products

YEAR 9 STAGE 5.2

WS Area diagrams

WS Binomial products

Expanding brackets

Expanding binomials

Expanding binomials

$(x + 4)$ and $(x - 7)$ are called **binomial expressions** because they each have exactly 2 terms.

$(x + 3)(x - 1)$ is called a **binomial product** because it is a product of 2 binomial expressions.

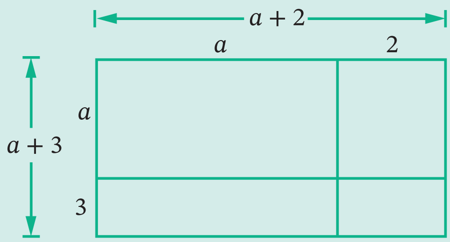
binomial = '2 terms'

Example 21

Expand $(a + 2)(a + 3)$ using an area diagram.

Solution

Draw an area diagram (rectangle) with length $(a + 2)$ and width $(a + 3)$ and divide the diagram into 4 smaller rectangles.



Foundation Standard Complex

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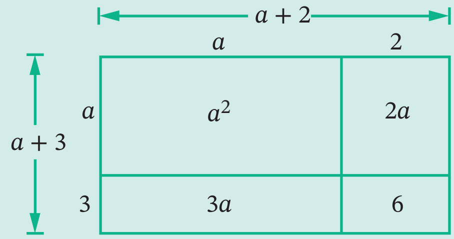
Find the area of each smaller rectangle.

Expand $(a + 2)(a + 3)$ by adding the areas of the 4 rectangles.



$(a + 2)(a + 3) = a^2 + 2a + 3a + 6$ Expanding

$= a^2 + 5a + 6$ Simplifying by collecting like terms.

Another way of expanding binomial products is to use the distributive law. Each term in the first binomial is multiplied by each term in the second binomial to give **4 terms**, which are collected and simplified.

YEAR 9 STAGE 5.2

3.13

Example 22

Expanding binomial products

Expand each binomial product.

a $(x + 5)(x + 9)$ **b** $(k + 3)(k - 7)$ **c** $(x - 6)(4x + 2)$

d $(3t - 1)(2t - 5)$ **e** $(a - 6)^2$

Solution

a $(x + 5)(x + 9) = x(x + 9) + 5(x + 9)$

$= x^2 + 9x + 5x + 45$ Expanding to make 4 terms.

$= x^2 + 14x + 45$ Simplifying.

b $(k + 3)(k - 7) = k(k - 7) + 3(k - 7)$

$= k^2 - 7k + 3k - 21$

$= k^2 - 4k - 21$

c $(x - 6)(4x + 2) = x(4x + 2) - 6(4x + 2)$

$= 4x^2 + 2x - 24x - 12$

$= 4x^2 - 22x - 12$

d $(3t - 1)(2t - 5) = 3t(2t - 5) - 1(2t - 5)$

$= 6t^2 - 15t - 2t + 5$

$= 6t^2 - 17t + 5$

e $(a - 6)^2 = (a - 6)(a - 6)$

$= a(a - 6) - 6(a - 6)$

$= a^2 - 6a - 6a + 36$

$= a^2 - 12a + 36$

YEAR 9 STAGE 5.2

EXERCISE 3.13 ANSWERS ON P. 552

Extension: Expanding binomial products **U F R C**

EXAMPLE 21

1 Expand each binomial product by copying and completing the area diagram.

a $(x + 2)(x + 7)$

	x	2
x	---	---
7	---	---

b $(x + 5)(x + 1)$

	x	5
x	---	---
1	---	---

2 Expand $(p + 6)(p + 2)$. Select the correct answer **A**, **B**, **C** or **D**.

A $p^2 + 12$ **B** $p^2 + 6p + 12$ **C** $p^2 + 8p + 8$ **D** $p^2 + 8p + 12$

EXAMPLE 22

3 Use the distributive law to expand each binomial product algebraically. **R**

a $(x + 1)(x + 4)$ **b** $(x + 6)(x + 3)$ **c** $(x + 5)(x + 4)$ **d** $(x + 2)(x + 2)$

e $(x + 2)(x + 6)$ **f** $(x + 10)(x + 3)$ **g** $(x + 8)(x + 1)$ **h** $(x + 7)(x + 7)$

i $(x - 1)(x + 4)$ **j** $(x - 3)(x + 2)$ **k** $(x + 1)(x - 4)$ **l** $(x - 5)(x + 6)$

m $(x - 4)(x - 5)$ **n** $(x + 9)(x - 3)$ **o** $(x - 5)(x - 3)$ **p** $(x - 7)(x - 7)$

4 **a** Expand each binomial product.

i $(x + 3)(x - 3)$ **ii** $(x + 4)(x - 4)$ **iii** $(x - 2)(x + 2)$ **iv** $(x - 7)(x + 7)$

b Can you see a simpler way of expanding these products? Explain how you would expand $(x + 8)(x - 8)$. **R** **C**

5 Expand each binomial product.

a $(x + 1)(2x + 4)$ **b** $(3x - 1)(x + 2)$ **c** $(2x + 3)(x + 1)$ **d** $(x - 3)(5x + 2)$

e $(4x - 3)(x - 2)$ **f** $(5x + 2)(2x - 3)$ **g** $(x - 3)(3x - 1)$ **h** $(4x + 7)(x - 3)$

6 Expand each binomial square.

a $(a + 5)^2$ **b** $(x + 2)^2$ **c** $(y + 7)^2$

d $(x - 3)^2$ **e** $(2a + 1)^2$ **f** $(3a - 5)^2$

7 Expand:

a $(a + b)^2$ **b** $(a + b)(a - b)$

8 Expand $\left(4x+\frac{1}{2}\right)^2$. Select **A**, **B**, **C** or **D**.

A $16x^2+4x+\frac{1}{4}$ **B** $16x^2+\frac{1}{4}$ **C** $4x^2+2x+\frac{1}{4}$ **D** $16x^2+2x+\frac{1}{4}$

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Power plus ANSWERS ON P. 552

ANSWERS ON P. 552

1 Simplify each expression.

a $-7p - 8q + 3p - 5q + 4q$ **b** $-m - 3n - 5m - 7mn$

c $3k - 2j - 5 - 4k + 6j + 9$ **d** $a^2 + 2b^2 + 3b^2 - a^2$

e $4x^2 + 6x - x^2 - 12x$ **f** $xy + yz + xz + 4xy + 3xz + yz$

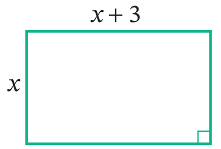
g $-d^2 + 7 + 6e^2 - 3d^2 - 2e^2 + 9$ **h** $5f^2 + 6f + 2 + 8f^2 - 1$

2 For each shape, write an algebraic expression for:

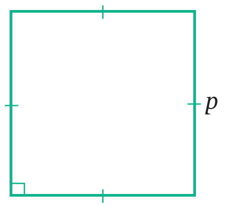
i its perimeter **ii** its area.

(Give your answer in simplest form.)

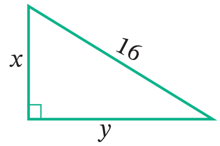
a



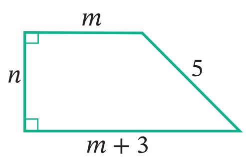
b



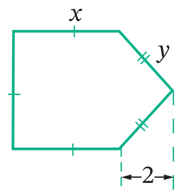
c



d



e



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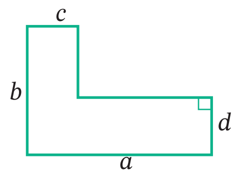


3 Expand and simplify each expression.

a $x(x - 3) + 5(x - 3)$ **b** $m(a + 4) + m(a - 7)$

c $-3(y - 4) + 7(y - 3)$ **d** $4(2a + b + 3c) - 2(a + 3b - c)$

e $p(p - 3q + 2r) + 2p(4r - 3p - 2q)$ **f** $7k^2 + 3(k^2 - 3k - 1) - (6k^2 - 4k + 2)$

g $12 - 5(4f - 6g - 3)$ **h** $5(2x^2 - 5x + 3) - 4(3x^2 - 8x - 25)$

i $(x + 3)(x + 5)$ **j** $(p - 1)(p + 10)$

4 Evaluate each expression if $p = 4$, $q = -6$, $r = 3$ and $t = -1$.

a $p + q$ **b** pq **c** $r^2 + t^2$ **d** $q^2 - 6p + rt$

e $\frac{pq}{rt}$ **f** $6 - 2r + p$ **g** $2p + 3q + 4r - 5t$ **h** $\frac{p+4}{2}$

i $\frac{q+r}{p+t}$ **j** $\frac{r+7}{5} - \frac{q-t}{5}$ **k** $\frac{p+3}{10} + \frac{r-2}{2}$ **l** $q + pr - qt$

5 Factorise each expression.

a $9pq + 5p - 2p^2$ **b** $44k - 33l - 11m$ **c** $2x - 7x^2 + 3xy$

d $x(y + 2) + 3(y + 2)$ **e** $a(b - 3) - 7(b - 3)$ **f** $4(k + 1) - j(k + 1)$

6 Simplify each expression.

a $\frac{m}{2} + \frac{m}{2}$ **b** $\frac{7n}{10} - \frac{3n}{10}$ **c** $\frac{2k}{3} + \frac{k}{3}$ **d** $\frac{3d}{5} - \frac{d}{5}$

e $\frac{5f}{8} + \frac{7f}{8}$ **f** $\frac{3c}{10} + \frac{4c}{10}$ **g** $\frac{4}{x} + \frac{2}{x}$ **h** $\frac{7}{a} + \frac{5}{b}$

3.13

Qz
Algebraic expressions

CHAPTER 3 REVIEW

Language of maths

base	brackets	collecting	consecutive
distributive law	evaluate	expand	expanded form
expression	factor	factorise	formula
highest common factor (HCF)	index laws	indices	like terms
power	pronumeral	substitute	term
unknown	unlike terms	variable	

1 Why does the word 'variable' have that name? Look up its meaning in the dictionary.

2 What are **unlike terms**?

3 How do you **expand** an algebraic expression?

4 What is the name given to the largest algebraic term that divides evenly into 2 or more given terms?

5 What is the opposite of **expanding** an algebraic expression?

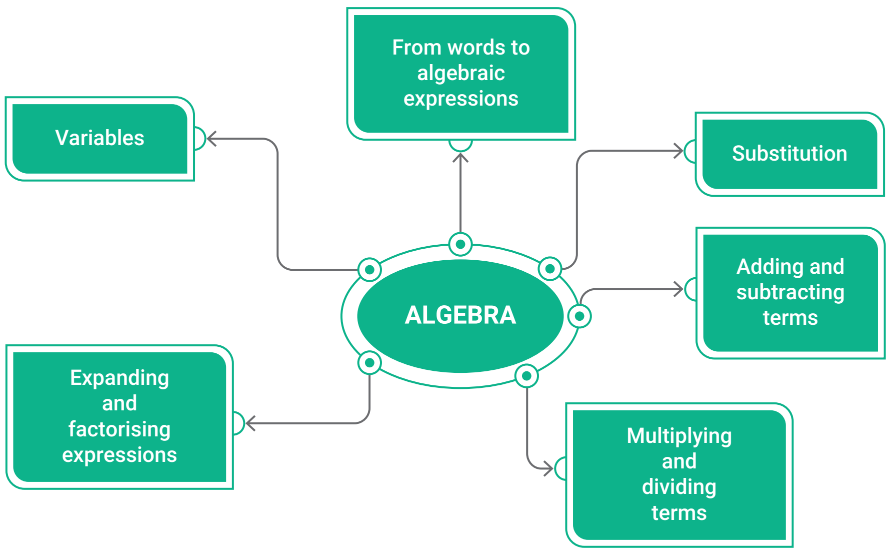
6 Look up some non-mathematical meanings of the word 'factor'. Use 'factor' in a sentence to show one of these meanings.

WS
Mind map: Algebra

Topic summary

- What parts of algebra did you remember from Year 7? What new rules have you learnt?
- Are there parts of this chapter that you still do not understand? Talk to your teacher.
- Give 2 examples of jobs where algebra is needed.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 3 ANSWERS ON P. 552

1 Simplify each expression. 3.01

- **a** $p \times 6$
- **b** $r \times (-1)$
- **c** $4 \times d \times g$
- **d** $t + t + t + t + t$
- **e** $14m + 6m$
- **f** $11k - 6k$
- **g** $6 \times y \times y$
- **h** $16 \div a$
- **i** $5 \times c \times 4 \times d$

2 Write each expression in expanded form. 3.01

- **a** cd
- **b** $3y^2$
- **c** $\frac{w}{6}$
- **d** $c^2 + y$

3 Write each statement as an algebraic expression. 3.02

- **a** 4 more than y
- **b** 6 times b minus 3
- **c** p divided by q

4 Evaluate $2n - p$ when $n = 11$ and $p = -3$. Select the correct answer **A**, **B**, **C** or **D**. 3.03

- **A** 19
- **B** 25
- **C** 208
- **D** 214

5 If $x = 5$ and $y = -2$, evaluate each expression. 3.03

- **a** $x + y$
- **b** xy
- **c** $3x - 2y$
- **d** $6y^2$
- **e** $6x \div y$
- **f** $x^2 + y^2$

6 Simplify each expression. 3.05

- **a** $5k + 7k$
- **b** $29d - 12d$
- **c** $4m + 18m - 5m$
- **d** $13m - 3m + 8m$
- **e** $4ac + 4ac - 5ca$
- **f** $11cd + 7cd - 9cd$
- **g** $7x + 3y + 3x + 7y$
- **h** $13r + 4s + 11s + 2r$
- **i** $33h + 6g - 5h + g$

7 Simplify each expression. 3.06

- **a** $5 \times 2x$
- **b** $2b \times 7d$
- **c** $11m \times 6$
- **d** $3p \times 8q$
- **e** $4 \times 2w \times 4w$
- **f** $15j \times 5$
- **g** $3m \times 6m \times 2$
- **h** $4p \times 3p \times 5p$

8 Simplify each expression. 3.07

- **a** $50x \div 2$
- **b** $28b \div 7d$
- **c** $66m \div 6$
- **d** $\frac{32rs}{4}$
- **e** $\frac{15p}{3p}$
- **f** $14d^2 \div 2$
- **g** $14e^2 \div 2e$
- **h** $13b^2 \div b$
- **i** $\frac{18x^2}{9x}$
- **j** $\frac{44m^2n^2}{11m}$
- **k** $\frac{16e^2g}{4eg}$
- **l** $\frac{35a^2e^2}{7ae}$

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3.09 **9** Expand and simplify each expression.

a $6(n + 2)$ **b** $-5(2b + 7)$

c $j(9j - 3)$ **d** $10 + 2(n + 4)$

e $22v - 3(v + 5)$ **f** $5(y + 2) + 4(y - 7)$

3.09 **10** Which expression is $5(c - d) - 2c + 3d$ simplified? Select **A**, **B**, **C** or **D**.

A $3c - 2d$ **B** $3c + 8d$ **C** $3c - 8d$ **D** $5c - 5d - 2c + 3d$

3.10 **11** Find the highest common factor of each pair of terms.

a $6a$ and 12 **b** $3p^2$ and $18p$ **c** $7t$ and 14

3.11 **12** Factorise each expression.

a $3m + 9$ **b** $5h - 35$ **c** $6a + 9ab$

d $10h - 12hm$ **e** $xy + xyz$ **f** $cd - de$

g $a^2 - 6a$ **h** $8kj + 24j^2$

3.12 **13** Factorise each expression.

a $-2 - 4q$ **b** $-4f - 16$ **c** $-12 + 15d$

d $-ab + b$ **e** $-16b - 24bd$ **f** $-4k^2 - k$

g $-x^2 + 7x$ **h** $-h^2i - hi^2$

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